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A Merton Model of Credit Risk with Jumps

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Abstract: In this note we consider a Merton model for default risk, where the firm's value is driven by a Brownian motion and a compound Poisson process.

Keywords: Merton model, default risk, default probability, processes with jumps

1 Introduction

Various models of Merton's type for credit risk have been studied so far (refer [1] to [8]). This paper aims to our recent results, where a model driven by a jump process is studied in [9] and another model governed by a jumps-diffusion is investigated in [10]. Suppose that the asset value V_t of a company, under a risk neutral measure, is given by the following differential equation

$$dV_t = (r-\beta\lambda)V_t dt + \sigma V_t dW_t + V_{t-} dQ_t, \tag{1.1}$$

where W_t is a standard Brownian motion, $Q(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process, $N(t)$ is a Poisson process with intensity $\lambda > 0$, Y_i's are independent and identically distributed random variables with $E(Y_i) = \beta$. All of these processes are supposed to be considered under the risk neutral measure. In (1.1), r is the interest rate, $\sigma > 0$ is a constant and N_t expresses the number of jumps of Q_t while Y_i is the i-th jump size of $Q(t)$.
The model (1.1) reflects a fact that, the firm's value can change randomly not only in a continuous way but also in a cumulatively discrete fashion.
We will study on the probability of default of the company when its value V_t is less than some debts.

2 Case of one debt L

A bankruptcy situation will occur at some time t when the company asset value is less than a debt L. And the problem is how to calculate the default probability $P(V_t < L)$.
It is known that the solution of (1.1) is given by (see [7])

$$V_t = V_0 \exp[\sigma W_t + (r-\beta\lambda - \frac{\sigma^2}{2})t] \prod_{i=1}^{N_t}(Y_i+1). \tag{2.1}$$

We see that

$$\ln V_t = \ln V_0 + \sigma W_t + (r-\beta\lambda - \frac{\sigma^2}{2})t + \sum_{i=1}^{N_t} \ln(Y_i+1).$$

And the event $\{V_t < L\}$ or $\{\ln V_t < \ln L\}$ means that

$$\sigma W_t + Z_t < x_t, \tag{2.2}$$

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where

$$x_t = \ln L - (r - \beta\lambda - \frac{\sigma^2}{2})t - \ln V_0, \tag{2.3}$$

$$Z_t = \sum_{i=1}^{N_t} U_i, \quad \text{with } U_i = \ln(1+Y_i). \tag{2.4}$$

Z_t is also a compound Poisson process where U_i's are i.i.d. random variables.
We calculate first the characteristic function $\Psi_{Z_t}(s)$ of Z_t. Z_t is also a compound Poisson process where U_i are i.i.d. random variables.
We recall first the characteristic function $\Psi_{Z_t}(s)$ of Z_t:

$$\begin{aligned}
\Psi_{Z_t}(s) &= E(e^{isZ_t}) \\
&= \sum_{j=0}^{\infty} E\left(e^{isZ_t}|N_t = j\right)P(N_t = j) \\
&= \sum_{j=0}^{\infty} E\left(e^{is(U_1+\ldots+U_j)}\right)P(N_t = j) \\
&= \sum_{j=0}^{\infty} \left(Ee^{isU_1}\ldots Ee^{isU_j}\right)P(N_t = j) \\
&= \sum_{j=0}^{\infty} (\psi_U(s))^j \frac{(\lambda t)^j}{j!}e^{-\lambda t} = \exp[\lambda t(\psi_U(s)-1)]
\end{aligned} \tag{2.5}$$

where $\psi_U(s)$ is the common characteristic function of U_i's.
It is known also that, for a compound Poisson process as Z_t we have $\mu(t) = EZ_t = \lambda tE(U_i) = \lambda tE\ln(1+Y_i) = \lambda tm$; $\sigma^2(t) = VarZ_t = \lambda tE(U_i^2) = \lambda tE[\ln(1+Y_i)]^2 = \lambda t\gamma^2$, where $E\ln(1+Y_i) = m$ and $E[(\ln(1+Y_i))^2] = \gamma^2$.

Denote by $\overline{Z}_t$ the normalization of Z_t

$$\overline{Z}_t = \frac{Z_t - \mu(t)}{\sigma(t)}.$$

And we will show that Z_t has an approximately normal distribution.
Indeed, according to the Taylor expansion for characteristic function

$$\psi_U(s) = \sum_{k=0}^{\infty} \frac{(is)^k}{k!}E|U|^k,$$

we can write

$$\psi_U(s) = 1 + ism - \frac{\gamma^2}{2}s^2 + o(s^2). \tag{2.6}$$

Now we compute the characteristic function of $\overline{Z}_t = \frac{1}{\sigma(t)}Z_t - \frac{\mu(t)}{\sigma(t)}$,

$$\Psi_{\overline{Z}_t}(s) = e^{-is\frac{\mu(t)}{\sigma(t)}}\Psi_{Z_t}(s/\sigma(t)).$$

Taking account of (2.5) and (2.6) we have

$$\begin{aligned}
\Psi_{\overline{Z}_t}(s) &= e^{-is\frac{\mu(t)}{\sigma(t)}}\exp[\lambda t(\psi_U(s/\sigma(t))-1)] \\
&= e^{-is\frac{\mu(t)}{\sigma(t)}}\exp[i\lambda tm\frac{s}{\sigma(t)} - \frac{\lambda t\gamma^2}{\sigma(t)^2}\frac{s^2}{2} + o(\frac{s^2}{t})] \\
&= e^{-is\frac{\mu(t)}{\sigma(t)}}\exp[is\mu(t)/\sigma(t) - \frac{\sigma^2(t)}{\sigma^2(t)}\frac{s^2}{2} + o(\frac{s^2}{t})] \\
&= \exp(\frac{-s^2}{2} + o(\frac{s^2}{t})), \quad \text{as } t \to \infty.
\end{aligned}$$

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Then $\overline{Z}_t \simeq \mathcal{N}(0,1)$ or $Z_t \simeq \mathcal{N}(\mu(t),\sigma(t)^2)$, where $\mu(t)=\lambda tE\ln(1+Y_i)$, $\sigma(t)=\sqrt{\lambda tE[\ln(1+Y_i)]^2}=\sqrt{\lambda t}\gamma$.
Now we can consider σW_t+Z_t as a sum of two independent normal random variables for each t large enough, so it has also a normal distribution with mean

$$\mu^*(t)=\mu(t)=\lambda tE\ln(1+Y_i)$$

and variance

$$\sigma^*(t)=\sigma^2+\sigma^2(t)=\sigma^2+\lambda tE[\ln(1+Y_i)]^2,$$

where $\sigma>0$ is a known constant as in (1.1).
And

$$P(\sigma W_t+Z_t<x_t)\approx\Phi(\frac{x_t-\mu^*(t)}{\sigma^*(t)}), \tag{2.7}$$

where $\Phi(x)$ is the standard normal distribution function.
We are now in the position to state the follow theorem.
Theorem 2.1 *The default probability can be approximated by*

$$P_{default}\approx\frac{1}{\sigma^*(t)\sqrt{2\pi}}\int_{-\infty}^{x_t}e^{-(u-\mu^*(t))^2/2\sigma^{*2}(t)}du, \tag{2.8}$$

where

$$x_t=\ln L-(r-\beta\lambda-\sigma^2/2)t-\ln V_0$$
$$\mu^*(t)=\lambda tE\ln(1+Y_i),\quad \sigma^*(t)=\lambda tE[\ln(1+Y_i)^2].$$

3 Case of many liabilities $L_1,L_2,\ldots,L_m$

Now we consider the case where the company faces up numerous debts $L_1,L_2,...,L_m$ that should be paid at times t_1, t_2,...,t_m respectively, with $t_1<t_2<...<t_m=T$.
The company will jump into default position before the time T if and only if at one of time t_i $(i=1,2,\ldots,m)$, it happens that

$$V_{t_i}<L_i.$$

So the probability of default before T is

$$P_{default}(0,T)=1-P(V_{t_i}>L_i,\forall t_i).$$

Denote $L=\max\{L_1,\ldots,L_m\}$ It is easy to see that for all $t_i(i=1,\ldots,m)$ we have

$$(V_{t_i}>L_i)\supset(V_{t_i}>L).$$

Then

$$P_{default}(0,T)\leq 1-P(V_{t_i}>L,\forall t_i). \tag{3.1}$$

Put $X_t=\sigma W_t+Z_t$, where, as before $Z_t=\sum_{i=1}^{N_t}U_i$, $U_i=\ln(1+Y_i)$. The inequality $V_{t_i}>L$ is equivalent to

$$X_{t_i}=\sigma W_{t_i}+Z_{t_i}>\ln L-\ln V_0-(r-\beta\lambda-\frac{\sigma^2}{2})t_i:=x_{t_i}.$$

Consider the event

$$A=\{V_{t_i}>L,\forall t_i\}=\bigcap_{i=1}^{m}\{X_{t_i}>x_{t_i}\}. \tag{3.2}$$

Then

$$P_{default}(0,T)\leq 1-P(A).$$

It is known that a compound Poisson process is a process of independent increments. The processes (W_t) and (Z_t) are independent and both are of independent increments, so is the process $X_t=\sigma W_t+Z_t$.
Denoting by A_i the event $\{X_{t_i}>x_i\},i=1,2,\ldots,m$ we can see that

$$A_1=\{X_{t_1}>x_{t_1}\}=\{X_{t_1}-X_0>x_{t_1}\},$$

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$$A_2 = \{X_{t_2} > x_{t_2}\} = \{X_{t_2} - X_{t_1} > x_{t_2} - X_{t_1}\} \supset \{X_{t_2} - X_{t_1} > x_{t_2} - x_{t_1}\},$$

if A_1 occurs.

$$\ldots$$

$$A_m = \{X_{t_m} > x_{t_m}\} = \{X_{t_m} - X_{t_{m-1}} > x_{t_m} - X_{t_{m-1}}\} \supset \{X_{t_m} - X_{t_{m-1}} > x_{t_m} - x_{m-1}\},$$

if $A_1, \ldots A_{m-1}$ occur.

Put $B_i = \{X_{t_i} - X_{t_{i-1}} > x_{t_i} - x_{t_{i-1}}\}$ for $i = 1, 2, \ldots, m$ and $x_0 = 0$ by convention. It follows that

$$\bigcap_{i=1}^{m} B_i \subset \bigcap_{i=1}^{m} A_i = A.$$

Because of the independence of increments we have

$$P(A) \geq P(\bigcap_{i=1}^{m} B_i) = \prod_{i=1}^{m} P(B_i), \tag{3.3}$$

And by definition of B_i,

$$\begin{aligned} P(B_i) &= P(X_{t_i} - X_{t_{i-1}} > x_{t_i} - x_{t_{i-1}}) \\ &= P(\sigma(W_{t_i} - W_{t_{i-1}}) + (Z_{t_i} - Z_{t_{i-1}}) > x_{t_i} - x_{t_{i-1}}). \end{aligned} \tag{3.4}$$

Put $\overline{X}_i = X_{t_i} - X_{t_{i-1}}$, $\overline{W}_i = \sigma(W_{t_i} - W_{t_{i-1}})$ and $\overline{Z}_i = Z_{t_i} - Z_{t_{i-1}}$, where Z_t is defined as in (2.4). The random variable $\overline{W}_i$ has normal distribution $\mathscr{N}(0, \sigma^2(t_i - t_{i-1}))$. The random variable $\overline{Z}_i = \sum_{k=N_{t_{i-1}}+1}^{N_{t_i}} U_k$ has the same distribution with that of $\sum_{k=1}^{N_{t_i - t_{i-1}}} U_k$ since U_i's are i.i.d and N_t is a process of stationary and independent increments.
We can see that the distribution of $\overline{Z}_i$ is given by

$$\begin{aligned} F_{\overline{Z}_i}(z) = P(\overline{Z}_i \leq z) &= \sum_{n=0}^{\infty} P(N_{t_i - t_{i-1}} = n) P(\overline{Z}_i \leq z / N_{t_i - t_{i-1}} = n) \\ &= \sum_{n=0}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda (t_i - t_{i-1})} P(\overline{Z}_i \leq z / N_{t_i - t_{i-1}} = n) \\ &= \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda (t_i - t_{i-1})} P(\sum_{k=1}^{n} U_i \leq z) \\ &= \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda (t_i - t_{i-1})} F_U^{*n}(z), \end{aligned} \tag{3.5}$$

where F_U^{*n} is the n fold convolution of common distribution of U_k's.
Suppose now that U_i's are continuous random variables, so are Z_i's and $\overline{Z}_i$'s. Then the density function of $\overline{X}_i = \overline{W}_i + \overline{Z}_i$ is

$$\begin{aligned} f_{\overline{X}_i}(x) = f_{\overline{W}_i} \star f_{\overline{Z}_i}(x) &= \int_{-\infty}^{\infty} f_{\overline{W}_i}(x - z) f_{\overline{Z}_i}(z) dz \\ &= \frac{1}{\sigma \sqrt{2\pi (t_i - t_{i-1})}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x - z)^2}{2\sigma^2 (t_i - t_{i-1})} \right] f_{\overline{Z}_i}(z) dz, \end{aligned} \tag{3.6}$$

where $f_{\overline{Z}_i}(z) = \frac{d}{dz} F_{\overline{Z}_i}(z)$ is the density function of $\overline{Z}_i$.
Now we have

$$P(B_i) = 1 - \int_{-\infty}^{x_{t_i} - x_{t_{i-1}}} f_{\overline{X}_i}(x) dx,$$

where $f_{\overline{X}_i}(x)$ is defined by (3.6).
And so, the following assertion is ready to be stated:

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Theorem 3.1 *If U_i's are continuous random variables then the probability of default before T is estimated by*

$$P_{default}(0,T) \le 1 - \prod_{i=1}^{m}\left(1 - \int_{-\infty}^{x_{t_i}-x_{t_{i-1}}}\left[\frac{1}{\sigma\sqrt{2\pi(t_i - t_{i-1})}}\times \right.\right.$$
$$\left.\left.\times \int_{-\infty}^{\infty} \exp\left[-\frac{(x-z)^2}{2\sigma^2(t_i - t_{i-1})}\right] f_{\overline{Z}_i}(z)dz\right]dx\right), \quad (3.8)$$

where

$$x_{t_i} = \ln L - \ln V_0 - (r - \beta\lambda - \frac{\sigma^2}{2})t_i \quad (3.9)$$

and

$$f_{\overline{Z}_i}(z) = \sum_{n=0}^{\infty} \frac{d}{dz}\frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} P(\overline{Z}_i \le z / N_{t_i - t_{i-1}} = n). \quad (3.10)$$

4 Particular cases of Theorem 3.1

We consider some particular cases for distribution of U_k's.

4.1. Case of normal random variables

Suppose that $U = U_k \sim \mathcal{N}(0,1)$ then we have $\sum_{k=1}^{n} U_k \sim \mathcal{N}(0,n)$ with density function $\frac{1}{\sqrt{2\pi n}} e^{-z^2/2n}$ and the density of $\overline{Z}_i$ is

$$f_{\overline{Z}_i}(z) = \frac{1}{\sqrt{2\pi n}} \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} e^{-z^2/2n} \quad (4.1)$$

From (3.8) and (4.1) we have

$$P_{default}(0,T) \le 1 - \prod_{i=1}^{m}\left(1 - \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}(t_i - t_{i-1})^n e^{-\lambda(t_i - t_{i-1})} \frac{1}{2\pi\sigma\sqrt{n(t_i - t_{i-1})}} \times \right.$$
$$\left.\times \int_{-\infty}^{x_{t_i}-x_{t_{i-1}}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x-z)^2}{2\sigma^2(t_i - t_{i-1})} - \frac{z^2}{2n}\right] dzdx\right). \quad (4.2)$$

4.2. Case of exponential random variable U_k with parameter $\nu > 0$

We know that if $U_k \sim \exp(\nu)$ then $\sum_{k=1}^{n} U_k \sim Gamma(n,\nu)$ with the density function

$$\frac{z^{n-1} e^{-z/\nu}}{\nu^n \Gamma(n)},$$

where Γ is Gamma function. Then

$$f_{\overline{Z}_i}(z) = \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} \frac{z^{n-1} e^{-z/\nu}}{\nu^n \Gamma(n)}.$$

We can see the estimation in (3.8):

$$P_{default}(0,T) \le 1 - \prod_{i=1}^{m}\left(1 - \int_{-\infty}^{x_{t_i}-x_{t_{i-1}}} \frac{1}{\sigma\sqrt{2\pi(t_i - t_{i-1})}} \int_{0}^{\infty} \exp\left[-\frac{(x-z)^2}{2\sigma^2(t_i - t_{i-1})}\right] \times \right.$$
$$\left.\times \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} \frac{z^{n-1} e^{-z/\nu}}{\nu^n \Gamma(n)} \right] dzdx\right)$$
$$= 1 - \prod_{i=1}^{m}\left(1 - \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}(t_i - t_{i-1})^n e^{-\lambda(t_i - t_{i-1})} \frac{1}{\sigma\sqrt{2\pi(t_i - t_{i-1})}} \times \right.$$
$$\left.\times \int_{-\infty}^{x_{t_i}-x_{t_{i-1}}} \int_{0}^{\infty} \exp\left[-\frac{(x-z)^2}{2\sigma^2(t_i - t_{i-1})} - \frac{z}{\nu}\right] \frac{z^{n-1}}{\nu^n \Gamma(n)} dzdx\right). \quad (4.3)$$

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5 When $U = U_k$'s are general discrete random variables

In this case we have

$$
\begin{aligned}
P(\overline{Z}_i = z) &= P(\sum_{k=1}^{N_{t_i-t_{i-1}}} U_k = z) = \sum_{n=1}^{\infty} P(N_{t_i-t_{i-1}} = n) P(\sum_{k=1}^{N_{t_i-t_{i-1}}} U_k = z / N_{t_i-t_{i-1}} = n) \\
&= \sum_{n=1}^{\infty} P(N_{t_i-t_{i-1}} = n) P(\sum_{k=1}^{n} U_k = z) \\
&= \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} P(\sum_{k=1}^{n} U_k = z).
\end{aligned} \tag{5.1}
$$

Denote by $\mathscr{L}$ the set of all possible values of $\overline{Z}_i \equiv^d \sum_{k=1}^{N_{t_i-t_{i-1}}} U_k$. So that

$$
\begin{aligned}
P(\overline{X}_i < x) &= P(\sigma \overline{W}_i + \overline{Z}_i < x) = \sum_{z \in \mathscr{L}} P(\sigma \overline{W}_i < x - z) P(\overline{Z}_i = z) \\
&= \sum_{z \in \mathscr{L}} \sum_{n=1}^{\infty} \int_{-\infty}^{x-z} \frac{1}{\sigma \sqrt{2\pi (t_i - t_{i-1})}} \exp[-\frac{u^2}{2\sigma^2 (t_i - t_{i-1})})] \times \\
&\times \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} P(\sum_{k=1}^{n} U_k = z) du.
\end{aligned} \tag{5.2}
$$

The default probability in this case is estimated by

$$
\begin{aligned}
P_{default}(0,T) \le 1 - \prod_{i=1}^{m} \Bigg(1 - \sum_{z \in \mathscr{L}} \sum_{n=1}^{\infty} \frac{1}{\sigma \sqrt{2\pi (t_i - t_{i-1})}} \int_{-\infty}^{x_{t_i} - x_{t_{i-1}}} \exp[-\frac{(x-z)^2}{2\sigma^2 (t_i - t_{i-1})}] dx \times \\
\times \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} P(\sum_{k=1}^{n} U_k = z) \Bigg).
\end{aligned} \tag{5.3}
$$

6 U is Poisson random variable with parameter $\beta > 0$

If $U = U_k \sim Poisson(\beta)$ then

$$\sum_{k=1}^{n} U_k \sim Poisson(n\beta)$$

with mass probability

$$p_z = P(\sum_{k=1}^{n} U_k = z) = e^{-n\beta} \frac{(n\beta)^z}{z!}, \;\; z = 0, 1, 2, \ldots$$

Then

$$
\begin{aligned}
P_{default}(0,T) &\le 1 - \prod_{i=1}^{m} \Bigg(1 - \sum_{z=0}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\sigma \sqrt{2\pi (t_i - t_{i-1})}} \times \\
&\times \int_{-\infty}^{x_{t_i} - x_{t_{i-1}}} \exp[-\frac{x^2}{2\sigma^2 (t_i - t_{i-1})} - n\beta] dx \frac{\lambda^n (t_i - t_{i-1})^n}{n!} e^{-\lambda(t_i - t_{i-1})} \frac{(n\beta)^z}{z!} \Bigg) \\
&= 1 - \prod_{i=1}^{m} \Bigg(1 - \sum_{z=0}^{\infty} \sum_{n=1}^{\infty} \frac{\lambda^n (t_i - t_{i-1})^n}{n!} \frac{(n\beta)^z}{z!} \frac{1}{\sigma \sqrt{2\pi (t_i - t_{i-1})}} \times \\
&\times \int_{-\infty}^{x_{t_i} - x_{t_{i-1}}} \exp[-\frac{x^2}{2\sigma^2 (t_i - t_{i-1})} - n\beta] dx \Bigg).
\end{aligned} \tag{6.1}
$$

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